

Direct calculation of the K -matrix for pion electro-production in the delta channel

P. Alberto¹, L. Amoreira², M. Fiolhais¹, B. Golli^{3,a}, and S. Širca⁴

¹ Department of Physics and Centre for Computational Physics, University of Coimbra, 3004-516 Coimbra, Portugal

² Department of Physics, University of Beira Interior, 6201-001 Covilhã, Portugal

³ Faculty of Education, University of Ljubljana and J. Stefan Institute, 1000 Ljubljana, Slovenia

⁴ Faculty of Mathematics and Physics, University of Ljubljana and J. Stefan Institute, 1000 Ljubljana, Slovenia

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Abstract. We present a method to calculate directly the K -matrix for the pion electro-production processes in the framework of chiral quark models which allows for a clean separation of the resonant amplitudes from the background. The method is applied to the calculation of the multipole amplitudes M_{1+} , E_{1+} , and S_{1+} in the Δ channel within the Cloudy Bag Model. A good overall description is found in a broad energy range.

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1 Introduction

Electro-production experiments reveal important information on the structure of nucleon resonances and provide stringent tests of quark models. In particular the $\Delta(1232)$ has been studied extensively (see [1] and references therein for a short review and basic nomenclature). In these studies ([2], see also [3,4] and [5] in the elastic sector) the important role of the pion cloud in baryons has become evident, manifesting itself in a relatively large probability for the quadrupole excitation of the Δ . Such a large probability cannot be explained in the framework of the constituent-quark model unless the exchange currents generated from the one-pion-exchange and/or the one-gluon-exchange potentials are included as required by current conservation [6]. This is also an indication of the relevance of pions or, equivalently, the $q\bar{q}$ -pairs.

In most approaches only the amplitudes for the excitation of the resonance have been calculated, treating it as a bound state, *i.e.* ignoring its decay. While such an approach can be justified in the case of weak or electromagnetic resonance decays, its use in the case of strongly decaying resonances is not well founded. In fact, the excited states manifest themselves as resonances in meson scattering and, since the resonant scattering (as well as the electro-production process) is always accompanied by non-resonant processes, the extraction of the resonant amplitudes is not straightforward. The resonant contribution is

related to the pole residue of the corresponding K -matrix; following the notation of [7], the K -matrix for scattering is parameterized as

$$K = \frac{C}{E_R - E} + D \quad (1)$$

and the K -matrix for the electro-production as

$$K = \frac{A}{E_R - E} + B, \quad (2)$$

where C and A represent the resonant parts, while D and B the background. E is the invariant mass of the system. In order to extract the resonant part of electro-production amplitudes of given multiplicities, information not only from electro-production but also from scattering is needed. In the model calculation of these amplitudes one usually takes the experimental values for the parameters of the resonance such as the position, the width, and the background phase shift. While this is possible in the case of the $\Delta(1232)$ where relatively precise measurements are available, such an approach cannot be used in the case of other resonances, *e.g.* the Roper resonance. The only sensible approach is therefore to calculate both electro-production and scattering within the same model.

The aim of this work is to construct a feasible computational scheme for the full electro-production amplitudes, calculating directly the pertinent K -matrices. The resulting matrices for scattering and electro-production appear

^a e-mail: bojan.golli@ijs.si

in the forms (1) and (2); to separate the resonant contribution from the background it is therefore sufficient to pick up the respective residues. From the K -matrices it is possible to deduce the electro-production amplitudes as a function of E , as well as their dependence on the photon virtuality Q^2 . Furthermore, the method is able to predict the EMR and CMR ratios not only at the K -matrix pole but also at the T -matrix pole which allows us to make the comparison with calculations based on the T -matrix.

We show that in models in which the pion field is linearly coupled to the quark core it is possible to construct a computational scheme which goes beyond the usual perturbation approach. We present the calculation for one such model, the Cloudy Bag Model in sect. 5. The amplitudes are sufficiently well reproduced from the pion threshold up to the energy region where the two-pion decay becomes important and the assumption of the single-pion channel breaks down. Our calculation of the M_{1+} amplitude is similar to that in ref. [8] using the T -matrix approach (see also [9]). However, to the best of our knowledge neither the full E_{1+} amplitude has been calculated in the framework of quark models, nor has the Q^2 -dependence of the amplitudes been explored away from the resonance.

2 Electro-production amplitudes in the K -matrix formalism

The K -matrix for πN scattering is defined as

$$K_{\beta\alpha} = -\pi \langle \Phi_\beta | H' | \Psi_\alpha^P \rangle = -\pi \langle \Psi_\beta^P | H' | \Phi_\alpha \rangle$$

(see, *e.g.*, [10]), where H' is the interaction part of the Hamiltonian, $|\Phi_\alpha\rangle$ are the asymptotic (unperturbed) states with α labeling the pion-nucleon system, and $|\Psi_\alpha^P\rangle$ are the *principal-value* states satisfying

$$|\Psi_\alpha^P\rangle = |\Phi_\alpha\rangle + \frac{\mathcal{P}}{E - H_0} H' |\Psi_\alpha^P\rangle, \quad (3)$$

and normalized as

$$\langle \Psi_\alpha^P(E) | \Psi_\beta^P(E') \rangle = \delta(E - E') \delta_{\alpha\beta} (1 + K^2)_{\alpha\alpha}.$$

The K -matrix is related to the familiar T -matrix¹ by

$$T = -\frac{K}{1 - iK}.$$

In the case of a single channel, the K -matrix is equal to the tangent of the πN scattering phase shift, $K = \tan \delta$.

In order to introduce the electro-production amplitudes in this formalism, we make the usual assumption that “switching on” the electro-magnetic interaction H_γ does not change the strong scattering amplitudes, *i.e.* the principal-value states (3) remain unchanged. The K -matrix for the electro-magnetic process is

$$K_{\gamma\pi} = -\pi \langle \Psi^P(m_s, m_t; \mathbf{k}_0, t) | H_\gamma | N(m'_s, m'_t); \mathbf{k}_\gamma, \mu \rangle.$$

¹ Here we use the definition of the T -matrix as, *e.g.*, in [7] which differs by a factor π from that in [10].

Here the initial state corresponds to the incoming virtual photon with four-momentum $(\omega_\gamma, \mathbf{k}_\gamma)$, $\omega_\gamma^2 - \mathbf{k}_\gamma^2 = -Q^2$ and polarization μ , and the nucleon with the third component of spin m'_s and isospin m'_t ; the final state consists of a nucleon and a scattered pion with four-momentum (ω_0, \mathbf{k}_0) and third component of isospin t . In the c.m. frame the nucleon momentum is opposite to the photon (pion) momentum, \mathbf{k}_γ , which defines the direction of the z -axis.

We expand the pion-nucleon states in a basis with good total angular momentum J and isospin T which we write as

$$\Psi_{JT}^P(M_J M_T; k_0, l) = K_{\pi\pi}^{JT} \tilde{\Psi}_{JT}(M_J M_T; k_0, l).$$

Here $K_{\pi\pi}^{JT}$ is the K -matrix for pion scattering in the channel JT and is related to the corresponding T -matrix by $T_{\pi\pi}^{JT} = K_{\pi\pi}^{JT} / (1 - iK_{\pi\pi}^{JT})$. The advantage of using $\tilde{\Psi}$ over Ψ^P is that it is a smooth function of the energy and its norm does not diverge at a (possible) resonance where $K \equiv \tan \delta \rightarrow \infty$. The incoming photon-nucleon state takes the form

$$|N(m'_s, m'_t); \mathbf{k}_\gamma, \mu\rangle = \sqrt{\omega_\gamma k_\gamma} a_\mu^\dagger(\mathbf{k}_\gamma) |N m'_s m'_t\rangle,$$

where $a_\mu^\dagger(\mathbf{k}_\gamma)$ is the creation operator for the photon and the factor $\sqrt{\omega_\gamma k_\gamma}$ ensures proper normalization.

In this article we study the production of p -wave pions in the Δ channel below the two-pion threshold, though the calculation can actually be extended to higher energies until the effect of the two-pion channel becomes prominent. For simplicity, we neglect the recoil corrections to the nucleon ground state. To obtain the electro-production amplitudes in this channel, we keep only the p -wave pions and the $J = T = \frac{3}{2}$ components in the expansion of the πN system (in this case we drop the JT superscripts). The T -matrix for electro-production can then be written as

$$T_{\gamma\pi} = \pi T_{\pi\pi} \frac{1}{\sqrt{2\pi}} \sum_m K_\lambda Y_{1m}(\hat{r}) C_{\frac{1}{2} m_s 1 m}^{\frac{3}{2} \lambda} C_{\frac{1}{2} \frac{1}{2} 1 0}^{\frac{3}{2} \frac{1}{2}}.$$

Here we have introduced the analogues of the familiar transverse helicity amplitudes:

$$K_\lambda = \sqrt{\omega_\gamma k_\gamma} \langle \tilde{\Psi}_\Delta(M_J = \lambda) | \frac{e_0}{\sqrt{2\omega_\gamma}} \int d\mathbf{r} \boldsymbol{\varepsilon}_\mu \cdot \mathbf{j}(\mathbf{r}) \times e^{i\mathbf{k}_\gamma \cdot \mathbf{r}} | N(m'_s = \lambda - \mu) \rangle, \quad (4)$$

where $\mathbf{j}(\mathbf{r})$ is the vector part of the electro-magnetic current and $M_T = m'_t = \frac{1}{2}$. The transverse electro-production amplitudes are

$$M_{1+}^{(3/2)} = -T_{\pi\pi} \sqrt{\frac{3}{16k_0 k_\gamma}} \frac{1}{2\sqrt{3}} (3K_{3/2} + \sqrt{3}K_{1/2}), \quad (5)$$

$$E_{1+}^{(3/2)} = T_{\pi\pi} \sqrt{\frac{3}{16k_0 k_\gamma}} \frac{1}{2\sqrt{3}} (K_{3/2} - \sqrt{3}K_{1/2}). \quad (6)$$

The scalar amplitude is

$$S_{1+}^{(3/2)} = T_{\pi\pi} \sqrt{\frac{3}{16k_0 k_\gamma}} \frac{1}{\sqrt{2}} K_S,$$

where

$$K_S = e_0 \sqrt{\frac{k_\gamma}{2}} \langle \tilde{\Psi}_\Delta(M_J = \frac{1}{2}) | \int d\mathbf{r} \rho(\mathbf{r}) e^{i\mathbf{k}_\gamma \cdot \mathbf{r}} | N(m'_s = \frac{1}{2}) \rangle. \quad (7)$$

The longitudinal amplitude L_{1+} is obtained by simply replacing the density operator by $\boldsymbol{\varepsilon}_0 \cdot \mathbf{j}(\mathbf{r})$.

The differential cross-section averaged over the initial states $m'_s = \pm \frac{1}{2}$ and $\mu = \pm 1$ reads

$$\frac{d\sigma_T}{d\Omega} = \frac{(2\pi)^4}{k_\gamma^2} \frac{1}{4} \sum_{m'_s \mu} \left| \frac{T_{\gamma\pi}}{\pi} \right|^2 \quad (8)$$

for the transverse photons; for the longitudinal photons the average is taken only over one polarization, $\mu = 0$. Equation (8) yields the familiar expression in terms of the pertinent electro-production amplitudes and the scattering angle (see, *e.g.*, [11]). The EMR and CMR ratios are defined in the usual way [12] as

$$\text{EMR} = \frac{\text{Re} [E_{1+}^{(3/2)*} M_{1+}^{(3/2)}]}{|M_{1+}^{(3/2)}|^2},$$

$$\text{CMR} = \frac{\text{Re} [S_{1+}^{(3/2)*} M_{1+}^{(3/2)}]}{|M_{1+}^{(3/2)}|^2}.$$

3 Calculation of the K -matrix in chiral quark models

In this work we consider quark models in which p -wave pions couple linearly to the three-quark core. Assuming a pseudo-scalar quark-pion interaction, the part of the Hamiltonian referring to pions can be written as

$$H_\pi = \int dk \sum_{mt} \left\{ \omega_k a_{mt}^\dagger(k) a_{mt}(k) + [V_{mt}(k) a_{mt}(k) + V_{mt}^\dagger(k) a_{mt}^\dagger(k)] \right\}, \quad (9)$$

where $a_{mt}^\dagger(k)$ is the creation operator for a p -wave pion with the third components of spin m and isospin t , and

$$V_{mt}(k) = -v(k) \sum_{i=1}^3 \sigma_m^i \tau_t^i \quad (10)$$

is the general form of the pion source, with $v(k)$ depending on the particular model.

Chew and Low [13] considered a similar model as (9) except that they did not allow for excitations of the nucleon core. They showed that the T -matrix for πN scattering is proportional to $\langle \Psi^{(-)}(E) | V_{mt}(k) | \Phi_N \rangle$, where $\Psi^{(-)}(E)$ are the incoming states. In general, the corresponding formula for the K -matrix cannot be written in such a simple form. However, in the JT basis, in which the K - and T -matrices are diagonal, it is possible to express

the K -matrix in the form²

$$K_{\pi\pi}^{JT}(k, k_0) = -\pi \sqrt{\frac{\omega_k}{k}} \langle \Psi_{JT}^P(E) | | V(k) | | \Phi_N \rangle. \quad (11)$$

The corresponding principal-value state obeys a similar equation as the in- and out-going states in the Chew-Low model:

$$| \Psi_{JT}^P \rangle = \sqrt{\frac{\omega_0}{k_0}} \left\{ [a^\dagger(k_0) | \Phi_N \rangle]^{JT} - \frac{\mathcal{P}}{H - E} [V(k_0) | \Phi_N \rangle]^{JT} \right\}, \quad (12)$$

where $[]^{JT}$ denotes coupling to good J and T . In order to rewrite this equation in a form more suitable for a practical calculation, we insert into (12) the complete set of eigenstates of H ,

$$\mathbf{1} = | \Phi_N \rangle \langle \Phi_N | + \sum_{JT} \int_{E_N + m_\pi}^\infty dE \frac{|\Psi_{JT}^P(E)\rangle \langle \Psi_{JT}^P(E)|}{1 + K_{\pi\pi}^{JT}(E)^2} + 2\pi\text{-states} + \dots$$

For energies below the 2-pion threshold only the one-pion states contribute, hence the equation of motion takes the form

$$| \Psi_{JT}^P(E) \rangle = \sqrt{\frac{\omega_0}{k_0}} [a^\dagger(k_0) | \Phi_N \rangle]^{JT} - \int dE' \frac{|\Psi_{JT}^P(E')\rangle \langle \Psi_{JT}^P(E')| [V(k_0) | \Phi_N \rangle]^{JT}}{1 + K(E')^2} \frac{1}{E' - E}. \quad (13)$$

Let us remark that for a general chiral quark model, the K -matrix and the corresponding principal-value state can be calculated variationally using the Kohn variational principle. For the single-channel scattering of a meson with momentum k_0 and energy ω_0 [14] it amounts to requiring the stationarity of

$$\tan \delta - \frac{\pi \omega_0}{k_0} \langle \Psi^P | H - E | \Psi^P \rangle,$$

where Ψ^P is a suitable chosen trial state.

4 Solution in the Δ channel

The important difference between our approach and the approach of Chew and Low is that the interaction $V(k)$ can generate bare quark states with quantum numbers different from the ground state by flipping the spin and isospin of the quarks. Furthermore, in the same spirit one can consider a more general type of models in which the quarks can be excited to higher spatial states. The state with the flipped spins plays a crucial role in the formation of the resonance in the delta channel. The general form (13), therefore, suggests the following ansatz in which we

² In the static approximation, k_0 is uniquely related to the energy $E = E_N + \omega_0$, so one can use either k_0 or E to label the states; for the on-shell K -matrix we write $K(k_0, k_0) = K(E)$.

separate the resonant quasi-bound state Φ_Δ from the state corresponding to pion scattering on the nucleon:

$$|\Psi_\Delta\rangle = \sqrt{\frac{\omega_0}{k_0}} \left\{ [a^\dagger(k_0)|\Phi_N\rangle]^{\frac{3}{2}\frac{3}{2}} + \int dk \frac{\chi(k, k_0)}{\omega_k - \omega_0} [a^\dagger(k)|\Phi_N^E(k)]^{\frac{3}{2}\frac{3}{2}} + c_\Delta^E |\Phi_\Delta\rangle \right\}. \quad (14)$$

We require that the resonant state Φ_Δ does not contain components with pions around the nucleon, since such a component is already included in the first two terms. We therefore impose the following constraint on Φ_Δ :

$$\langle \Phi_\Delta | a_{n\pi}^\dagger(k) | \Phi_N \rangle = 0.$$

We allow for the modification of the pion cloud in the nucleon in the presence of the scattering pion but require that such a state, $|\Phi_N^E\rangle$, asymptotically goes over to the true ground state $|\Phi_N\rangle$. The pion amplitude is related to the K -matrix by

$$\chi(k_0, k_0) = \frac{k_0}{\pi\omega_0} K_{\pi\pi}(k_0, k_0). \quad (15)$$

Iterating (13) using the ansatz (14) we obtain the solution for $\chi(k, k_0)$ in the form

$$\chi(k, k_0) = -c_\Delta^E \mathcal{V}_{\Delta N}(k) + \mathcal{D}(k_0, k).$$

The $\mathcal{V}_{\Delta N}(k)$ and $\mathcal{D}(k_0, k)$ obey the integral equations

$$\begin{aligned} \mathcal{V}_{\Delta N}(k) &= V_{\Delta N}(k) + \int \frac{dk'}{\omega'_k - \omega_0} \mathcal{K}_N(k, k') \mathcal{V}_{\Delta N}(k'), \\ \mathcal{D}(k_0, k) &= \mathcal{K}_N(k_0, k) + \int \frac{dk'}{\omega'_k - \omega_0} \mathcal{K}_N(k, k') \mathcal{D}_N(k_0, k'), \end{aligned}$$

where

$$V_{\Delta N}(k) = \langle \Phi_\Delta | |V(k)| | \Phi_N \rangle,$$

and $\mathcal{K}_N(k_0, k)$ is the kernel involving scattering channels also for $JT \neq \frac{3}{2}\frac{3}{2}$. It is dominated by the crossed term involving the nucleon; the contributions from the crossed terms involving the delta and the Roper resonance are small, while the channels with $J \neq T$ negligible. Neglecting the widths of the resonances (see discussion in appendix A) as well as assuming $\Phi_N^E \approx \Phi_N$ allows us to write the kernel in the form

$$\begin{aligned} \mathcal{K}_N(k', k) &= \frac{4}{9} \frac{\langle \Phi_N | |V(k')| | \Phi_N \rangle \langle \Phi_N | |V(k)| | \Phi_N \rangle}{\omega_k + \omega'_k - \omega_0} \\ &+ \frac{1}{36} \frac{\langle \Phi_N | |V(k')| | \Phi_\Delta \rangle \langle \Phi_N | |V(k)| | \Phi_\Delta \rangle}{\omega_k + \omega'_k + \varepsilon_\Delta - \omega_0} \\ &+ \frac{4}{9} \frac{\langle \Phi_N | |V(k')| | \Phi_R \rangle \langle \Phi_N | |V(k)| | \Phi_R \rangle}{\omega_k + \omega'_k + \varepsilon_R - \omega_0}. \quad (16) \end{aligned}$$

Here $\varepsilon_\Delta = E_\Delta - E_N$ and $\varepsilon_R = E_R - E_N$ are the delta-nucleon and the Roper-nucleon energy splittings, respectively.

The solution for c_Δ^E can be written as

$$[E_\Delta(\omega_0) - E] c_\Delta^E = -\mathcal{U}_{\Delta N}(k_0)$$

with

$$E_\Delta^\Delta = \langle \Phi_\Delta | H | \Phi_\Delta \rangle$$

and

$$\begin{aligned} \mathcal{U}_{\Delta N}(k_0) &= V_{\Delta N}(k_0) + \int \frac{dk}{\omega_k - \omega_0} V_{\Delta N}(k) \mathcal{D}_N(k_0, k), \\ E_\Delta(\omega_0) &= E_\Delta^\Delta - \Sigma_\Delta(\omega_0) = E_\Delta + \Sigma_\Delta(\varepsilon_\Delta) - \Sigma_\Delta(\omega_0), \\ \Sigma_\Delta(\omega_0) &= \int \frac{dk}{\omega_k - \omega_0} \mathcal{V}_{\Delta N}(k) V_{\Delta N}(k), \end{aligned}$$

where $E_\Delta = E_\Delta(\omega_0 = \varepsilon_\Delta)$ is the position of the pole (of the K -matrix). In a practical calculation we can always adjust a model parameter (*e.g.* the bare Δ energy) such that E_Δ corresponds to the experimental value.

The final result for the K -matrix, in which the resonant and the background contributions are explicitly separated, is

$$\begin{aligned} K_{\pi\pi}(E) &= \tan \delta = \pi \frac{\omega_0}{k_0} \chi(k_0, k_0) \\ &= \pi \frac{\omega_0}{k_0} \left[\frac{\mathcal{U}_{\Delta N}(k_0) \mathcal{V}_{\Delta N}(k_0)}{E_\Delta(\omega_0) - E} + \mathcal{D}(k_0, k_0) \right]. \end{aligned}$$

Having obtained the parameters of the scattering state (14), the calculation of the electro-production amplitudes is straightforward. In the type of models we are considering here, the current and the charge density operators can be split into quark and pion parts:

$$\mathbf{j}(\mathbf{r}) = \bar{\psi} \gamma (\frac{1}{6} + \frac{1}{2} \tau_0) \psi + i \sum_t t \pi_t(\mathbf{r}) \nabla \pi_{-t}(\mathbf{r}), \quad (17)$$

$$\rho(\mathbf{r}) = \bar{\psi} \gamma_0 (\frac{1}{6} + \frac{1}{2} \tau_0) \psi - i \sum_t t \pi_t(\mathbf{r}) P_{-t}^\pi(\mathbf{r}), \quad (18)$$

where P^π stands for the canonically conjugate pion field. The procedure used to calculate the matrix elements of (4) and (7) is sketched in appendix A.

5 Results for the Cloudy Bag Model

We shall investigate the capability of the method by calculating the electro-production amplitudes M_{1+} , E_{1+} , and S_{1+} in the resonant $J = T = \frac{3}{2}$ channel in the framework of the Cloudy Bag Model. The Hamiltonian of the model has the form (9) and (10) with

$$v(k) = \frac{1}{2f_\pi} \frac{k^2}{\sqrt{12\pi^2\omega_k}} \frac{\omega_{\text{MIT}}^0}{\omega_{\text{MIT}}^0 - 1} \frac{j_1(kR)}{kR},$$

where $\omega_{\text{MIT}}^0 = 2.0428$. The free parameters are the bag radius R and the energy splitting between the bare nucleon and the bare delta. For each R , we adjust the splitting such that the experimental position of the resonance is reproduced.

It is a known drawback of the model that the width of the delta is underestimated, irrespectively of the bag radius, if the pion decay constant f_π is fixed to the experimental value. By reducing f_π from 93 MeV to 83 MeV $> f_\pi > 78$ MeV we are able to reproduce the experimental

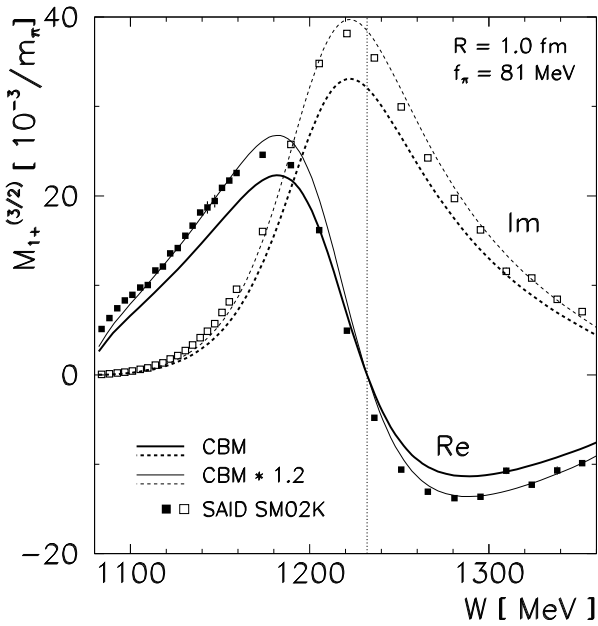


Fig. 1. The $M_{1+}^{(3/2)}$ electro-production amplitude in the CBM by using $R = 1.0$ fm and $f_{\pi} = 81$ MeV (thick curves), and multiplied by 1.2 (thin curves). The data points in the figures are the single-energy values of the SM02K (2 GeV) solution of the SAID π N partial-wave analysis [15].

phase shift in the energy range from the threshold to $E \sim 1300$ MeV for 0.8 fm $< R < 1.1$ fm. Since our aim here is to explore the applicability of the method to calculate a wide range of baryon properties as measured in pion production experiments, rather than to accurately reproduce particular experimental results, we have not attempted to further adjust the parameters of the model. We keep $R = 1.0$ fm and $f_{\pi} = 81$ MeV as the standard parameter set. Fitting the calculated phase shift with the ansatz (1) we get $C = \frac{1}{2}\Gamma = 58$ MeV and $D = \tan \delta_b = -0.42$, where δ_b is the background phase shift. The inclusion of the Roper in (16) contributes less than 5% to the width. Taking into account the finite widths of the delta and the Roper resonances in the evaluation of the sum over intermediate states (see appendix A) has a negligible effect on the results.

The dominant magnetic contribution calculated from (5) is shown in fig. 1. The reason why the experimental values are underestimated lies in a too weak γ N Δ vertex. In this model it is proportional to the isovector magnetic moment. For the nucleon its value is typically 20% lower than the experimental value, almost irrespectively of the model parameters [16]. Increasing the calculated amplitude by 20% we obtain an almost perfect agreement with the experiment throughout the energy range.

Regarding the E_{1+} amplitude, we encounter the well-known problem (see, *e.g.*, [17]) of large cancellations of terms in the expression for the electro-magnetic current, which leads to unreliable results. Instead, we use current conservation and calculate E_{1+} from the charge operator. The energy dependence of the real and imaginary parts (fig. 2) shows the correct pattern compared to the experiment, though the calculated magnitude is too small. The

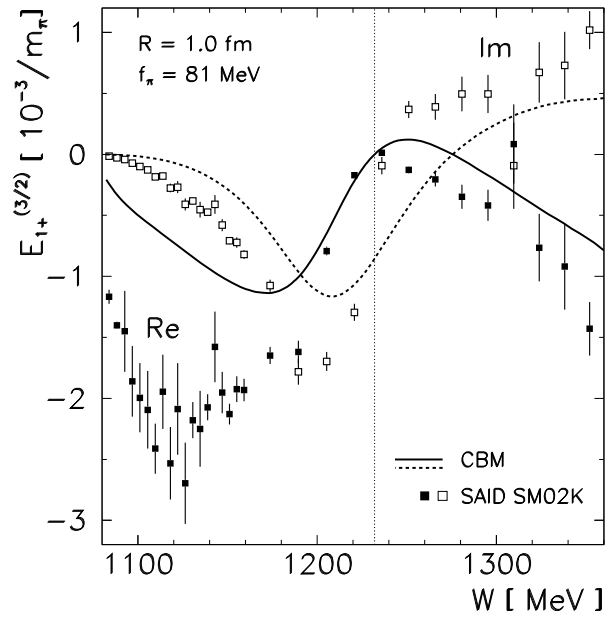


Fig. 2. The $E_{1+}^{(3/2)}$ electro-production amplitude in the CBM by using $R = 1.0$ fm and $f_{\pi} = 81$ MeV.

agreement is worse at low energies, although the corresponding experimental uncertainties are large as well.

Since in the K -matrix approach we can extract the pure resonance contribution at the pole of the K -matrix (this would not be possible if we worked with the T -matrix), we can directly compare our results with the calculation of the transition form factors G_{M1} and G_{E2} at the photon point within the same model [18]. We have explicitly checked that after substituting our matrix elements of V_{mt} in eqs. (A.3) and (A.4) by the corresponding bare values, the results of ref. [18] are consistent with ours. However, while in their calculation of G_{M1} it was possible to reproduce the experimental value by reducing the bag radius—and hence increasing the strength of the π qq vertex—this mechanism does not improve the agreement in the case of the M_{1+} amplitude. The reason is that increasing the strength of the quark-pion interaction leads to a larger width of the resonance, and since \sqrt{T} appears (implicitly) in the denominator of the amplitudes (4) and (7), M_{1+} decreases.

In the ratio of the E_{1+} and M_{1+} multipoles (the EMR), the influence of the too weak γ N Δ coupling is strongly reduced, and the agreement with the experiment above $E \simeq 1150$ MeV is much better (fig. 3).

In general, the Q^2 -dependence of the amplitudes is not well reproduced in the model, partly due to the rather peculiar form of $v(k)$ at large k . Figure 4 shows the energy dependence of the CMR for two non-zero values of Q^2 compared to SAID [15] and MAID [19] results based on rather scarce experimental data. Our calculation reproduces the general pattern, though the magnitude at the resonance and above it is not well reproduced.

From our results it is possible to extract the resonance parameters at the pole of the T -matrix, based on the separation of the amplitude into the resonant and

Table 1. Resonance pole parameters extracted from the computed $E_{1+}^{(3/2)}$ and $M_{1+}^{(3/2)}$ multipoles, compared to various determinations from data. The moduli r are in units of $10^{-3}/m_\pi$.

$R(\text{fm})/f_\pi(\text{MeV})$	r_E	ϕ_E	r_M	ϕ_M	R_Δ
1.1 / 78	0.95	-160°	16	-35°	$-0.034 - 0.047 i$
1.0 / 81	0.95	-165°	16	-38°	$-0.035 - 0.047 i$
0.9 / 83	0.97	-165°	16	-40°	$-0.036 - 0.049 i$
Ref. [20]	1.23	-154.7°	21.16	-27.5°	$-0.035 - 0.046 i$
Ref. [21], MSP fit	1.12	-162°	20.75	-36.5°	$-0.040 - 0.047 i$
Ref. [22], Fit 1	1.22	-149.7°	22.15	-27.4°	$-0.029 - 0.046 i$
Ref. [23], Fit A	1.38	-158°	20.9	-31°	$-0.040 - 0.053 i$

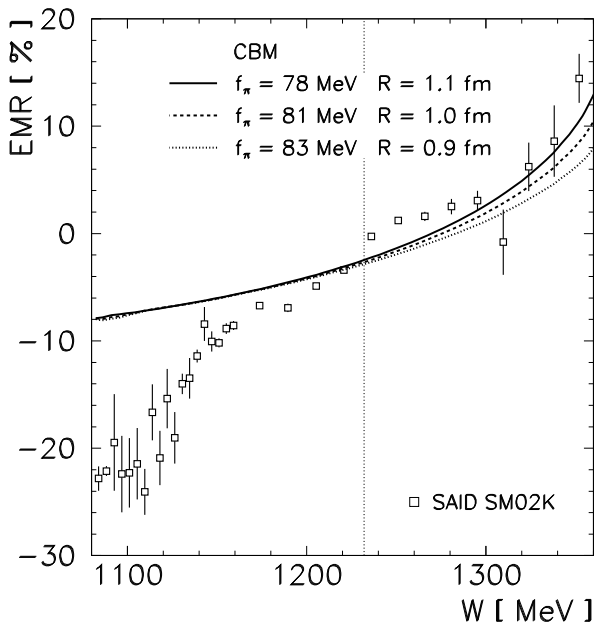


Fig. 3. The energy dependence of $\text{EMR} = \text{Re}[E_{1+}^{(3/2)*} M_{1+}^{(3/2)}] / |M_{1+}^{(3/2)}|^2$ at the photon point in the CBM, for three sets of model parameters.

background parts, *i.e.* $T = T_R + T_B$ using the parameterization [20,21] $T_R = r\Gamma_R e^{i\phi} / (M_R - E - i\Gamma_R/2)$. The parameters can be expressed in terms of A , B , C , and D which are determined by fitting our results to (1) and (2). Since the parameters of our model were chosen in order to reproduce the phenomenological phase shift, it is not surprising that the pole of the T -matrix appears at $E_R = M_R - i\Gamma_R/2 = (1211 - 49i)$ MeV which is almost exactly at the correct position $(1210 - 50i)$ MeV [24]. The corresponding moduli and phases for the transverse multipoles are shown in table 1. While the magnitudes are underestimated, the ratio as well as the phases are much better reproduced.

6 Summary and conclusions

We have investigated a method to calculate directly the K -matrices of resonant electro-production processes in the framework of chiral quark models. The main advantage of

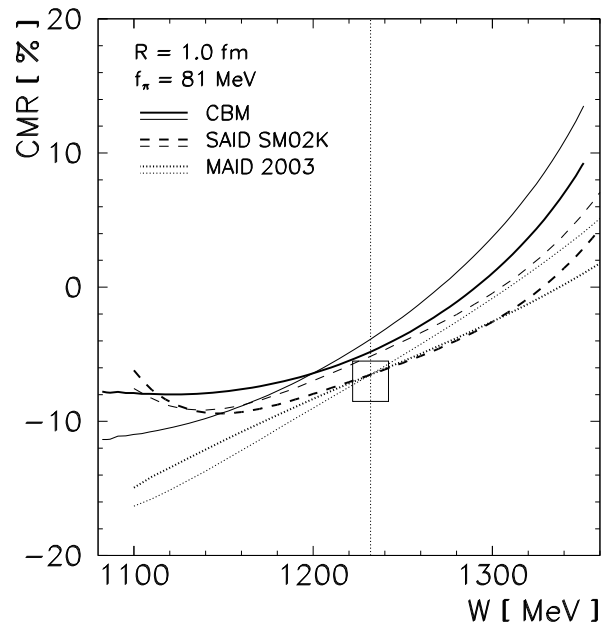


Fig. 4. The energy dependence of $\text{CMR} = \text{Re}[S_{1+}^{(3/2)*} M_{1+}^{(3/2)}] / |M_{1+}^{(3/2)}|^2$ at $Q^2 = 0.1$ (thin curves) and $0.5 (\text{GeV}/c)^2$ (thick curves) in the CBM compared to the results of SAID and MAID. The experimental CMR in the $\Delta E \simeq 10$ MeV vicinity of the Δ -resonance is $\simeq (-7.0 \pm 1.5)\%$ for $0.1 \leq Q^2 \leq 0.9 (\text{GeV}/c)^2$ [12,25] (rectangle).

the method shows up in the treatment of resonant channels in which the resonant part of the amplitude can be separated from the background part in an unambiguous way. Furthermore, the finite width of the resonance can be correctly taken into account.

The method has been successfully applied to the calculation of amplitudes in the Δ channel in the Cloudy Bag Model. In spite of the simplicity of the model we have been able to reproduce reasonably well the behavior of all amplitudes from the threshold up to the energies where the two-pion production becomes important. The method can be applied to other models with more sophisticated description of quark dynamics which so far have not been used outside the resonance peak.

In the future we intend to apply the method to the calculation of electro-production amplitudes in other channels. Particularly interesting is the Roper channel, where

the interplay between the resonant part induced by the excited quark core and the background due to the scattering pion being attached to the nucleon as well as to the delta, becomes crucial.

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Appendix A. Evaluation of matrix elements

In models in which the pions are linearly coupled to the quark source it is possible to derive some general relations for the matrix elements, independent of the particular quark model. Let us first note that if $|\Psi_A\rangle$ is an eigenstate of the Hamiltonian (9) then

$$(\omega_k + H - E_A)a_{mt}(k)|\Psi_A\rangle = -V_{mt}^\dagger(k)|\Psi_A\rangle, \quad (\text{A.1})$$

$$\begin{aligned} (\omega_k + \omega'_k + H - E_A)a_{mt}(k)a_{m't'}(k')|\Psi_A\rangle = \\ - \left[V_{mt}^\dagger(k)a_{m't'}(k') + V_{m't'}^\dagger(k')a_{mt}(k) \right] |\Psi_A\rangle. \end{aligned} \quad (\text{A.2})$$

The renormalization of the operator $\sum_{i=1}^3 \sigma_m^i \tau_t^i$ which appears in the quark parts of the EM currents (see (17) and (18)) takes the form

$$\langle \Psi_\Delta^P | \sum_{i=1}^3 \sigma^i \tau^i | \Phi_N \rangle = \frac{\langle \Psi_\Delta^P | V(k_0) | \Phi_N \rangle}{v(k_0)} = -\sqrt{\frac{k_0}{\omega_0}} \frac{K_{\pi\pi}(E)}{\pi v(k_0)}$$

where we have used (10), (11) and (15).

The pion contribution in (17) and (18) involves two-pion operators. To illustrate the procedure, let us consider the case of two creation operators: using the conjugate of (A.2) and inserting the complete set of states we can write

$$\begin{aligned} \langle \tilde{\Psi}_\Delta(E) | a_{mt}^\dagger(k) a_{m't'}^\dagger(k') | \Phi_N \rangle = \\ - \frac{\langle \Phi_N | V_{m't'}^\dagger(k') | \Phi_N \rangle \langle \Phi_N | a_{mt}(k) | \tilde{\Psi}_\Delta(E) \rangle}{(\omega_k + \omega'_k - \omega_0)} \\ - \sum_{JT} \int \frac{dE'}{1 + K_{\pi\pi}^{JT}(E')^2} \frac{\langle \Phi_N | V_{m't'}^\dagger(k') | \tilde{\Psi}_{JT}(E') \rangle}{(\omega_k + \omega'_k - \omega_0)} \\ \times \langle \tilde{\Psi}_{JT}(E') | a_{mt}(k) | \tilde{\Psi}_\Delta(E) \rangle - (k, m, t) \leftrightarrow (k', m', t'). \end{aligned} \quad (\text{A.3})$$

Again, the transition matrix elements involving $a(k)$ and $V(k)$ can be related to the K -matrix, *e.g.*,

$$\langle \Phi_N | a(k) | \Psi_\Delta^P(E) \rangle = \delta(k - k_0) - \frac{\langle \Phi_N | V^\dagger(k) | \Psi_{JT}^P(E) \rangle}{(\omega_k - \omega_0)}, \quad (\text{A.4})$$

hence

$$\langle \Phi_N | a(k) | \tilde{\Psi}_\Delta(E) \rangle = K_{\pi\pi}^{-1} \delta(k - k_0) - \frac{1}{\pi} \sqrt{\frac{k_0}{\omega_0}} \frac{\chi(k, k_0)}{(\omega_k - \omega_0) \chi(k_0, k_0)}.$$

The expression $K_{\pi\pi}^{JT}(E')^2 / (1 + K_{\pi\pi}^{JT}(E')^2) = \sin^2 \delta_{JT}$ is proportional to the cross-section in the P_{JT} channel and can be evaluated either from the calculated or the experimental phase shift. It yields sizable contributions only close to possible resonances (*e.g.* the delta and the Roper resonances). Furthermore, for a sufficiently narrow resonance at $E' = E_*$, this expression can be substituted by $\frac{1}{2} \pi \Gamma \delta(E' - E_*)$ leading to a similar expression as in the perturbation theory. As a consequence, the matrix element $\langle \tilde{\Psi}_{JT}(E') | a_{mt}(k) | \tilde{\Psi}_\Delta(E) \rangle$ in the last term substantially contributes only for $JT = \frac{3}{2} \frac{3}{2}$ and $E' \approx E_\Delta$.

A similar procedure is used to extract the one-pion amplitude around the bare delta below the 2-pion threshold:

$$\begin{aligned} \langle \Delta | a(k) | \Psi_\Delta(E) \rangle = - \int_{E_N + m_\pi}^\infty dE' \sqrt{\frac{\omega'_0}{k'_0}} \frac{K_{\pi\pi}^{JT}(E')^2}{1 + K_{\pi\pi}^{JT}(E')^2} \\ \times \frac{\langle \tilde{\Psi}_\Delta(E') | V^\dagger(k) | \tilde{\Psi}_\Delta(E) \rangle}{(\omega_k + E' - E)}. \end{aligned}$$

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